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**Modified Warshall** (All Pair Shortest Path)

# Theory :

The Floyd-Warshall algorithm, named after its creators Robert Floyd and Stephen Warshall, is a fundamental algorithm in computer science and graph theory. It is used to find the shortest paths between all pairs of nodes in a weighted graph. This algorithm is highly efficient and can handle graphs with both positive and negative edge weights, making it a versatile tool for solving a wide range of network and connectivity problems.

# Algorithm :

step 1 :

Initialize the solution matrix same as the input graph matrix as a first step. Step 2 :

Then update the solution matrix by considering all vertices as an intermediate vertex.

Step 3 :

The idea is to pick all vertices one by one and updates all shortest paths which include the picked vertex as an intermediate vertex in the shortest path.

Step 4 :

When we pick vertex number k as an intermediate vertex, we already have considered vertices {0, 1, 2, .. k-1} as intermediate vertices.

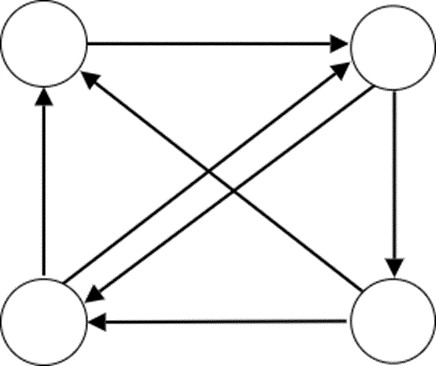
Step 5 :

For every pair (i, j) of the source and destination vertices respectively, there are two possible cases.

1 .k is not an intermediate vertex in shortest path from i to j. We keep the value of dist[i][j] as it is.

2. k is an intermediate vertex in shortest path from i to j. We update the value of dist[i][j] as dist[i][k] + dist[k][j], if dist[i][j] > dist[i][k] + dist[k][j]

**Problem :** Find the all pair shortest path for the following graph (source vertex is 1)

3

1 **2**

-4

4 5

2 12

**4 3**

-1

**Solution :**

# A0 :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | ∞ | ∞ |
| 2 | ∞ | 0 | 12 | 5 |
| 3 | 4 | ∞ | 0 | -1 |
| 4 | 2 | -4 | ∞ | 0 |

Find the matrix for vertex 1

A1 considering 1 as an intermiadiatary vertex

# A1 :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | ∞ | ∞ |
| 2 | ∞ | 0 | **12** | **5** |
| 3 | 4 | **7** | 0 | **-1** |
| 4 | 2 | **-4** | ∞ | 0 |

1) A0 [2,3] A0 [2,1] + A0 [1,3]

12 < ∞ + ∞

2) A0 [2,4] A0 [2,1] + A0 [1,4]

5 < ∞ + ∞

3) A0 [3,2] A0 [3,1] + A0 [1,2]

∞ 4 + 3

∞ > 7

4) A0 [3,4] A0 [3,1] + A0 [1,4]

-1 < 4 + ∞

5) A0 [4,2] A0 [4,1] + A0 [1,2]

-4 2 + 3

-4 < 5

6) A0 [4,3] A0 [4,1] + A0 [1,3]

∞ = 2 + ∞

Find the matrix for vertex 2

A2 considering 2 as an intermiadiatary vertex

# A2 :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | ∞ | ∞ |
| 2 | ∞ | 0 | **12** | **5** |
| 3 | 4 | **7** | 0 | **-1** |
| 4 | 2 | **-4** | ∞ | 0 |

|  |  |  |
| --- | --- | --- |
| 1) A1 [1,3] | A1 [1,2] + A1 [2,3] | |
| ∞ | 3 + | 12 |
| ∞ > | 15 |  |
| 2) A1 [1,4] | A1 [1,2] + A1 [2,4] | |
| ∞ | 3 + 5 | |
| ∞ > | 8 | |

3) A1 [3,1] A1 [3,2] + A1 [2,1]

4 < 7 + ∞

4) A1 [3,4] A1 [3,2] + A1[ 2,4]

-1 < 7 + 5

5) A1 [4,1] A1 [4,2] + A1 [2,1]

2 < -4 + ∞

6) A1 [4,3] A1 [4,2] + A1 [2,3]

∞ -4 + 12

∞ > 8

Find the matrix for matrix of vertex 3

A3 considering 3 as an intermiadiatary vertex

# A3 :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | 5 | 8 |
| 2 | 16 | 0 | 12 | 5 |
| 3 | 4 | 7 | 0 | -1 |
| 4 | 2 | -4 | 8 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1) | A2 [1,2] | |  | A2 [1,3] + A2 [3,2] | | | |
|  | 3 | |  | < 15 + 7 | | | |
| 2 ) | A2 [1,4] | |  | A2 [1,3] + A2[ 3,4] | | | |
|  | 8 | | < | 15 + (-1) | | | |
| 3) | A2 [2,1] | |  | A2 [2,3] + A2 [3,1] | | | |
| ∞ | | |  | | 12 | + | 4 |
| ∞ | | | > | | 16 |  |  |
| 4) | A2 | [2,4] |  | A2 [2,3] + A2 [3,4] | | | |
|  |  | 5 |  | 12 + (-1) | | | |
|  |  | 5 | > | 11 | | | |
| 5) | A2 | [4,1] |  | A2 [4,3] + A2 [3,1] | | | |
|  |  | 2 | < | 8 + 4 | | | |
| 6) | A2 | [4,3] |  | A2 [4,2] + A2 [2,3] | | | |
|  |  | -4 | < | 8 + 7 | | | |

Find the matrix for matrix of vertex 4

A4 considering 4 as an intermiadiatary vertex

A4 :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | 5 | 8 |
| 2 | 14 | 0 | 12 | 5 |
| 3 | 1 | 3 | 0 | -1 |
| 4 | 2 | -4 | 8 | 0 |

1) A4 [1,2] A4 [1,4] + A4 [4,2]

|  |  |  |  |
| --- | --- | --- | --- |
| 3 |  | 8 + | 4 |
| 3 | < | 12 |  |
| [1,3] |  | A4 [1,4] | + A4 [ 4,3] |
| 5 |  | 8 + | 8 |
| 5 | < | 16 |  |

2 ) A4

|  |  |
| --- | --- |
| 4) A4 [2,3]  12  12  5) A4 [3,1]  4  4  6) A4 [3,2]  7 | A4 [2,4] + A4 [4,3]  5 + 8  < 13  A4 [3,4] + A4 [4,1]  -1 + 2  > 1  A4 [3,4] + A4 [4,2  -1 + 4 |
|  |  |
|  |  |
|  |  |
|  |  |
| 7 | > 3 |

|  |  |  |  |
| --- | --- | --- | --- |
| 3) A4 [2,1] | A4 [2,4] + A4 [4,1] | | |
| 16 |  | 12 + | 2 |
| 16 | > | 14 |  |

]